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A Point-wise Solution for the Magnetic Field Vector

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Abstract

Magnetometers offer measurements that define the angle to the magnetic field. Knowledge of this restricts the axis of interest to lie on a cone centered on the magnetic field direction with an angle proportional to the measurement of magnetic field strength. With this restriction, formulae are derived for finding the relationship between the magnetic field pointing vector, the spin axis, and the magnitude of the magnetic field. Methods based on both calibrated and uncalibrated magnetometers are discussed. These formulae are also useful for the calibration of magnetometers.

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Contents

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1. Introduction

Inertial navigation systems (INS) are being developed for projectiles and rockets. These systems have higher spin rates than the traditional INS. In these applications, the largest errors are associated with the orientation or attitude of the body. Attitude estimation is vital to inertial navigation. Errors in attitude result in accelerometers being misaligned with respect to the INS reference frame. Within an INS, this misalignment will lead to accelerometer integration in the wrong directions and in time, will lead to large errors in position. Angular rate sensors can be integrated to estimate a body's attitude. Errors attributable to bias and random walk cause these estimates to drift away from the actual values. Also, angular rate sensors are sensitive to vibration. A method to obtain an independent measurement of an aspect of attitude could aid this process through the formulation of a Kalman filter or allow the engineer to bypass angular rate sensors and obtain a direct estimate of attitude. Knowledge of the attitude of a body's spin axis will improve the performance of an INS.

Magnetometers offer measurements that define the angle to the magnetic field. Knowledge of this angle restricts the axis of interest to lie on a cone centered on the magnetic field direction with an angle proportional to the measurement of magnetic field strength. With this restriction, formulae are derived for finding the relationship between the magnetic field pointing vector, the spin axis, and the magnitude of the magnetic field. Methods based on calibrated and uncalibrated magnetometers are discussed. These formulae are also useful for the calibration of magnetometers.

2. Background

Conley and Patton [1] discuss a method to find the spin axis of a sounding rocket via a solar sensor and one magnetometer. Their method uses iterative nonlinear least squares, Euler angles, and inner products to find the orientation of the spin axis. The inner product of two normalized vectors is the cosine of the angle between them. This method requires at least one spin cycle of data to implement.

Harkins and Hepner [2] discuss a method to find the orientation of the spin axis by the use of solar and magnetometer information. We can find the angle between the spin axis and the magnetic field by using the zero crossings of two magnetometers. This method requires at least one spin cycle to implement.

Harkins and Hepner named their magnetometer configuration a MAGSONDE (MAGnetic SONDE). A MAGSONDE is a device that uses two magnetometers to obtain estimates of the angle to the magnetic field. One magnetometer is in a plane that is orthogonal to the spin axis; the other is in the plane formed by the spin axis and the magnetometer orthogonally to the spin axis.

The magnetic coordinate system is formed by the spin axis of the projectile, a vector orthogonal to it in the same plane as the spin axis and the vector defining the direction to the magnetic field, and a third vector orthogonal to the previous two. If the spin axis and the magnetic field vectors are known, then the coordinate system can be defined via cross products. The first dimension is the spin axis; the second is the cross product of the third dimension and the spin axis; the third is the normalized cross product of the spin axis and magnetic field. A magnetometer in this frame aligned orthogonally to the spin axis will read zero when it is aligned with the third dimension and will give a maximum reading when aligned with the second dimension.

Since the magnetic field direction is assumed to be known, the accuracy to which the spin axis can be estimated will determine system performance. Finding the angle to the magnetic field gives one restriction on the spin axis; this angle defines a cone about the magnetic field vector upon which the spin axis must lie. The use of this with a second constraint will define the spin axis. For a projectile, the magnetic coordinate system will change during the flight because of the change in the orientation of the spin axis. The magnetic roll angle is the rotation about the spin axis, measured so that when the magnetometer output is zero and the output is increasing, the measure of the angle is zero.

3. Point-wise Solutions

Formulae are developed to find the angle between the magnetic field and the spin axis at any point of time. The accuracy of these equations can be assessed through simulations. Software was developed to implement the simulations and to find the theoretical performance of the formulae during noise-induced perturbations. These performance levels yield an ideal performance level that can only be approached in practice.

The idea is to form a ratio of the signals and eliminate the magnitude factor or to form a linear combination that will allow extraction of desired signal information. The background information for this report is contained in Harkins and Hepner [2], who use zero crossings of the magnetometers to find the direction to the magnetic field.

Sensor 1 is aligned orthogonally to the spin axis. The measurement from Sensor 1 is

$$M_{s1} = |M| \sin\sigma_m \sin\phi_s, \tag{1}$$

in which σ_m is the angle between the spin axis and the magnetic field, ϕ_s is the roll angle (about the spin axis), and M is the magnitude of the magnetic field. If Sensor 2 is at an angle of λ to the spin axis in the same plane as Sensor 1, then the measurement from Sensor 2 is

$$M_{s2} = \cos\lambda |M| \cos\sigma_m + \sin\lambda |M| \sin\sigma_m \sin\phi_s. \tag{2}$$

If the magnitude of the first term is greater than the largest magnitude of the second term, this signal will not cross zero. If angles are chosen randomly, then 50% of the time, Measurement 2 will not contain zero crossings. For each of these measurements, if the magnitude of the magnetic field is known, then each measurement can be thought of as the inner product of the normalized magnetometer axis and the magnetic field direction. Forming the ratio of these two measurements makes the solution amplitude independent. After simplification, the following result is obtained.

$$\frac{M_{s2}}{M_{s1}} = \frac{\cos\lambda \cot(\sigma_m)}{\sin\phi_s} + \sin\lambda \tag{3}$$

The two measurements and the angle of Sensor 2 (λ) are known, so the known quantities are moved to the right-hand side.

$$\frac{\cot(\sigma_m)}{\sin\phi_s} = \frac{1}{\cos\lambda}\left(\frac{M_{s2}}{M_{s1}} - \sin\lambda\right) \tag{4}$$

The sine of the magnetic roll angle can be found by reference to the zero crossings of Sensor 1. It will be the time since the last zero crossing divided by the time between zero crossings multiplied by 2π. This isolates the cotangent function; thus, we can find the angle to the magnetic field by using the arctangent function. For obvious reasons, this solution will not be stable as the value of Sensor 1 approaches zero. Away from that region, it should be possible to calculate the angle to the magnetic pointing vector with Equation (4). In the region where Sensor 1 is near zero, the inverse ratio could be used to find the angle to the magnetic field.

$$\tan\sigma_m = \frac{\cos\lambda}{\sin\phi_s}\left(\frac{M_{s1}}{M_{s2} - M_{s1}\sin\lambda}\right) \tag{5}$$

However, the factor involving the sine of the magnetic roll angle will not permit a solution as it approaches zero as the output from Sensor 1 does. An error

analysis of Equation (4) will provide regions of stable solutions for certain noise levels.

Software was designed to assess the sensitivity of the solution. At the indicated roll angles, the variance of the solution when each signal had Gaussian noise with standard deviations of 0.01 to 0.07 radian is depicted in Figure 1.

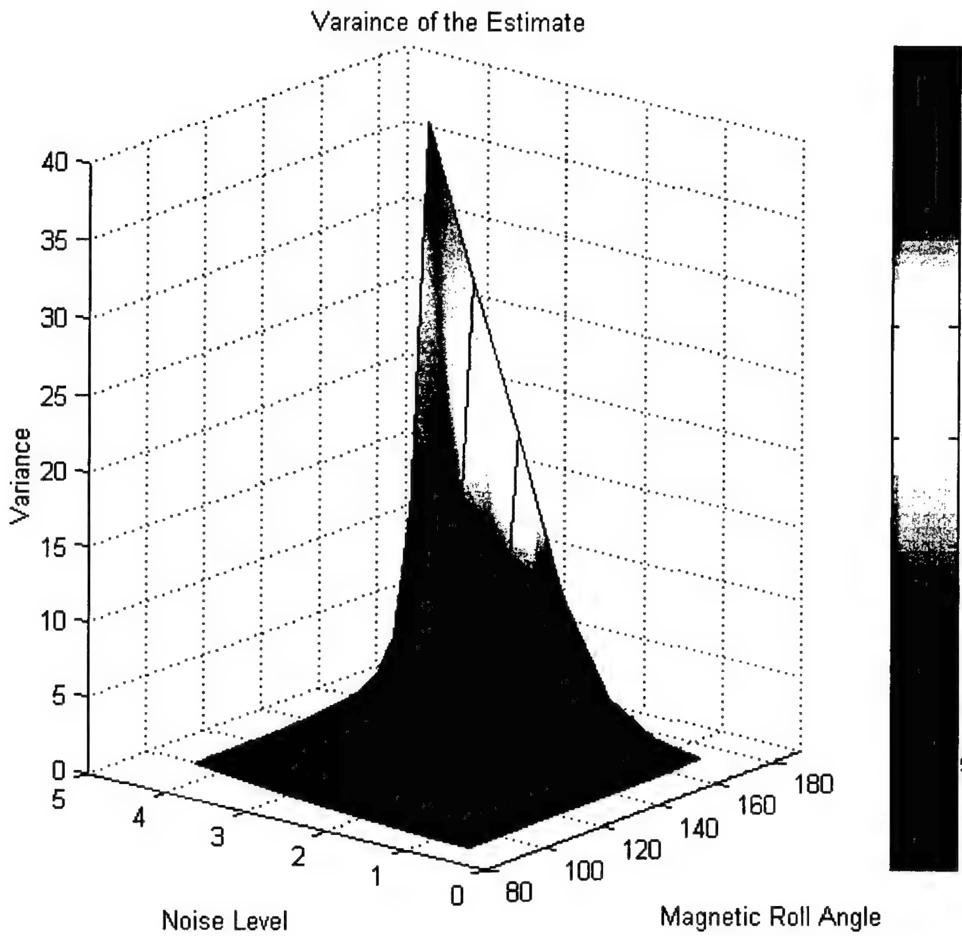

Figure 1. Estimation Variance as a Function of Noise and Angle.

The solution starts with a roll angle of 180 degrees and ends with a roll angle of 172 degrees. The solution is symmetrical about 90 degrees. As expected, the solution should be most stable for roll angles near 90 degrees (about 1.5 radians). At a noise level of 0.01 radian (0.57 degree), the solution's standard deviation for roll angles of 3 radians (172 degrees) is on the order of 5 degrees. This decreases to about 1.5 degrees when the roll angle is near 90 degrees.

A linear model will not fit well for magnetic roll angles within 0.5 radian (29 degrees) of 0 and 180 degrees. For roll angles not in this range, when the noise level is increased, a linear relationship between the standard deviation of

the input noise in radians and the variance of the solution is observed for the noise levels of 0.01 to 0.07 radian. Figure 1 clearly shows that the curvature increases with noise level and proximity to 180- or 0-degree roll angles.

Another method to find the pointing angle to the magnetic field can be devised by the use of the measurement from Sensor 2. When Sensor 1 has zero output, recall that Equation (4) is not appropriate for this situation. This is at a roll angle of 0 or 180 degrees in the magnetic coordinate system. Using Equation (2), we can find

$$\sigma_m = a\cos\left(\frac{M_{s2}}{|M|\cos(\lambda)}\right) \tag{6}$$

In this situation, the strength of the magnetic field needs to be known. When Sensor 1 measures zero, the magnitude of Sensor 2 defines the direction to the magnetic field. For example, if the output from sensor 2 were 0, this would indicate that the vector to the magnetic field is 90 degrees from the spin axis. The value of Sensor 2 indicates how far from orthogonal the magnetic field vector is pointing. Note that this formula would allow the calculation of field strength if the direction to the magnetic field were known or could be estimated; thus, it could be used for calibration.

The information available if Sensor 2 is zero can also be used to find the direction to the magnetic field. The assumption that the output is zero allows Equation (2) to be solved for the angle to the magnetic field.

$$\sigma_M = a\tan\left(\frac{-\cot(\lambda)}{\sin(\phi_s)}\right). \tag{7}$$

This solution requires the roll angle in the magnetic coordinate system to be found. Note: The solution is determined by the magnetic roll angle and this is equivalent to a timing parameter. Based on the assumption that the roll rate is constant, the amount of time between zeros corresponds to a portion of the rotational cycle. For Sensor 1, the time between zeros corresponds to 180 degrees. If the Sensor 2 measurement is greater than the Sensor 1 measurement, then the zero crossings will occur between 180 and 360 degrees. The ratio of the zero crossings will give the portion of 180 degrees where Sensor 2 was less than zero. Subtracting this ratio from 1 will give the portion of 180 degrees before the first zero and after the second. To find the magnetic roll angle at the first zero of Sensor 2, one can use the following equation.

$$\phi_s = \pi + \pi(1 - \text{ratio})/2. \tag{8}$$

This formula can be used to find the angle to the magnetic field, based on the ratio of each sensor's time durations between zeros. The magnetic roll angle at the first zero of the cycle can be found, based on the ratio and then inserted into the formula to obtain the angle to the magnetic field. This result in the Equation (7) yields the angle to the magnetic field.

Figure 2 shows the performance of this method of solution for 10,000 replications. Note that in this diagram, both the noise level and the standard deviation are in radians; the conversion factor to degrees is 57.296. Since the value of the ratio is equivalent to knowledge of the magnetic roll angle, the performance of Equation (7) depends on both the accuracy of the measurement of the roll angle and the actual value of the magnetic roll angle. Harkins and Hepner **[2]** discuss another method of using zero crossings to solve for the spin axis angle to the magnetic field.

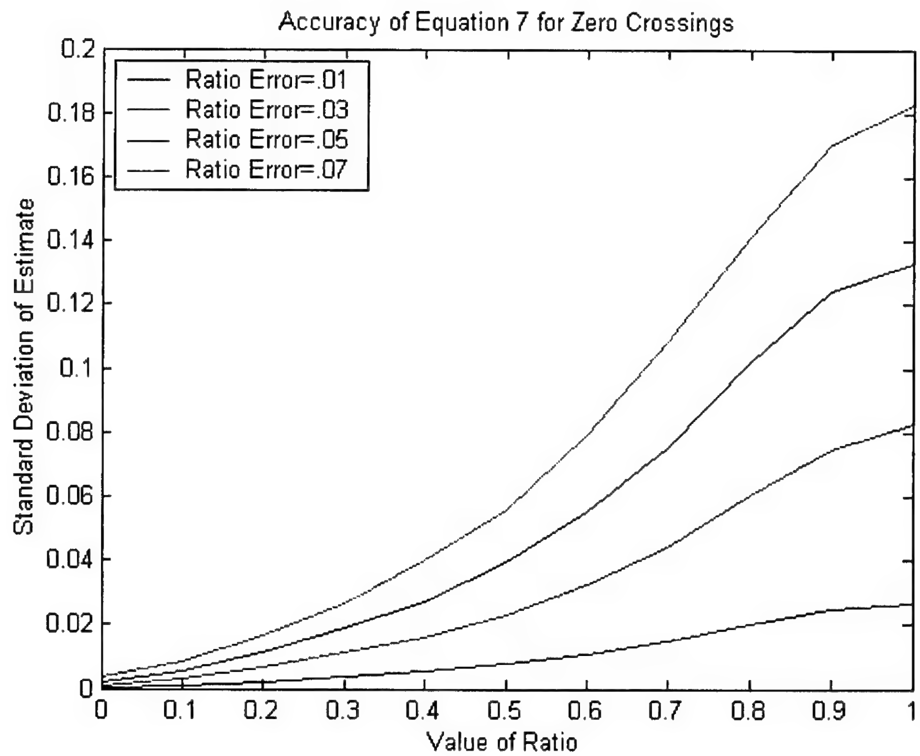

Figure 2. Sensitivity of Equation 7.

A variation of Equation (7) can be found by the use of the output of Sensor 1 as input to Equation (2) when the reading is zero. Solving for the desired angle yields

$$\sigma_m = a\cos\left(\frac{-M_{s1}}{|M|}\tan[\lambda]\right). \tag{9}$$

Use of this requires the knowledge of the perceived magnetic field strength.

The separation between the measurements and the magnetic roll angle contains the necessary information to determine the angle between the magnetic field and the spin axis. A modification of the difference between the two measurements can be used. One can find the measure of the angle between the spin axis and magnetic field by multiplying the value of Sensor 1 by the sine of the angle of sensor offset and then subtracting this value from the output of Sensor 2. The result is

$$\sigma_m = a\cos\left[\frac{M_{s2} - \sin(\lambda)M_{s1}}{\cos(\lambda)|M|}\right]. \qquad (10)$$

This result does not require knowledge of the magnetic roll angle; however, the strength of the magnetic field needs to be ascertained. Note that Equation (10) is the same as Equation (9) if Measurement 2 is 0. The fidelity of the formula will be determined by the accuracy of the measurements and the accuracy in the estimate of magnetic field strength. As the argument of the arc cosine function approaches 1, errors in the measurement and magnitude terms can drive the argument to be greater than 1. With an input noise of 0.57 degree for both measurements and the magnitude, the standard deviation of the solution was on the order of 1 degree for magnetic vectors between 50 and 130 degrees. Figure 3 shows the relationship based on a simulation; 10,000 replications were run for each magnetic field angle.

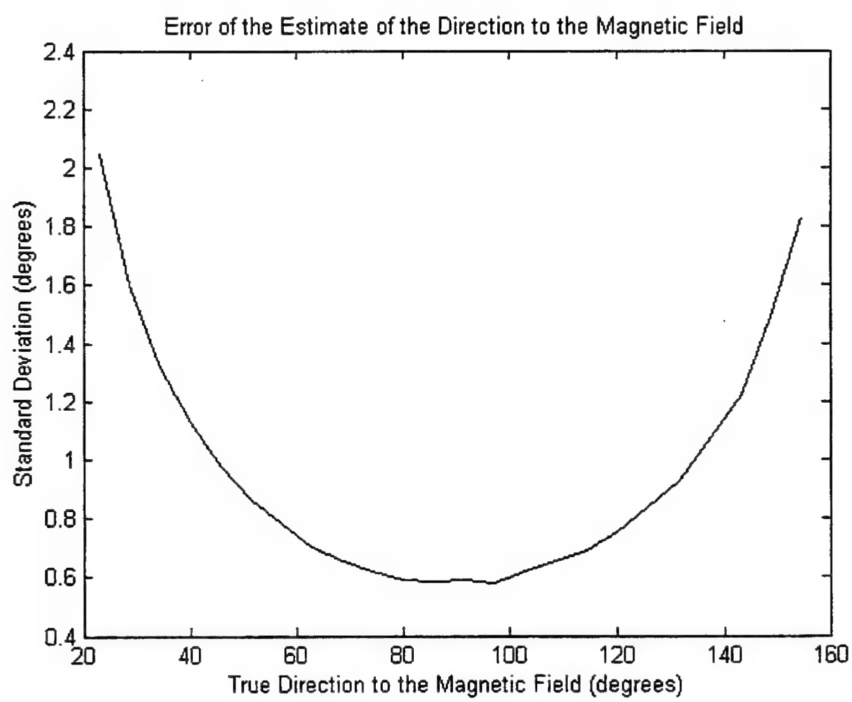

Figure 3. Sensitivity of Equation 10.

As with the first function for magnetic pointing angle, the errors are greatest at 0 and 180 degrees where the sine of the magnetic roll angle is zero. This simply indicates that as the spin axis and the direction to the magnetic field become collinear, Equation 10 loses its fidelity or usefulness.

4. Complete Solutions

In this section, a method to find the orientation of the spin axis is discussed. The attitude of a vector can be found, given the angles between the vector and two known directions. Of interest is the situation when the vector to be found is the spin axis and the known directions are the magnetic field and the direction to the sun or the solar vector. Each known direction and angle will yield a cone of possibilities in three-dimensional space; the intersection of the two cones will yield two possible directions or points. If a magnetometer is being used, it is possible to select one point, based on the time history of the magnetometer output. For one point, the output should be increasing; for the other, the output should be decreasing.

An Euler sequence is used to represent the attitude of one coordinate system (e.g., a body-fixed system) in terms of another (such as the INS reference system). The Euler aerospace sequence is one of the 12 possible Euler sequences for relating coordinate systems. The first rotation is about the Z-axis (ψ), followed by a rotation about the new Y-axis (θ), and the final rotation takes place about the newest X-axis (ϕ). When the aerospace sequence is used, the final X-axis is typically the spin axis of the body. For spinning projectiles, the angular rate about the spin axis is typically constant over a spin cycle. The validity of this assumption is critical to the method discussed. In addition, it is assumed that the magnetic field vector is known. Conley and Patton **[1]** use these two assumptions to develop their attitude estimator. The magnetometer output is proportional to the projection of the magnetic field onto the magnetometer. By describing the magnetometer in terms of Euler angles, we can develop a set of equations by considering the output to be the inner product of the magnetic field vector with the magnetometer orientation vector. With these equations, a method can be developed to find the desired Euler angles. Although Conley and Patton used the satellite ephemeris Euler sequence, their method can be recast via the aerospace sequence.

Next, assume that a magnetometer is placed orthogonally to the spin axis. Let the body-fixed coordinate system be defined so that the spin axis is the X-axis and the magnetometer is along the Y-axis. Starting with a reference coordinate system, the transition matrix to the body-fixed coordinate system (based on the aerospace sequence) is where the periods indicate the end of the term.

$$\begin{pmatrix} \cos\psi\cos\theta. & \sin\psi\cos\theta. & -\sin\theta. \\ \begin{matrix}\cos\psi\sin\theta\sin\phi - \\ \sin\psi\cos\phi.\end{matrix} & \begin{matrix}\cos\psi\cos\phi + \\ \sin\psi\sin\theta\sin\phi.\end{matrix} & \cos\theta\sin\phi. \\ \begin{matrix}\sin\psi\sin\phi + \\ \cos\psi\sin\theta\cos\phi.\end{matrix} & \begin{matrix}-\cos\psi\sin\phi + \\ \sin\psi\sin\theta\cos\phi.\end{matrix} & \cos(\theta)\cos(\phi). \end{pmatrix}$$

For a rotation matrix, the inverse and transposed matrices are equivalent. The X-axis in body-fixed coordinates can be represented by the vector (1,0,0). Transforming this to the reference coordinate system yields the first row as the vector result. The spin axis can be represented in terms of the Euler angles. The inner product of the spin axis (from the first row) and the direction of the magnetic field will yield an equation containing Euler angles that could be useful. The magnetometer is along the Y-axis, which corresponds to the second row of the matrix. The inner product of the Y-axis with the normalized direction to the magnetic field will equal the cosine of the angle between the two. From this matrix, the spin axis and two orthogonal vectors perpendicular to the spin axis can be found in terms of Euler angles.

Returning to Equation (1), we find that the cosine of the angle between the magnetometer and the magnetic field is equivalent to multiplying the sine of the angle between the magnetic field and the spin axis by the sine of the magnetic roll angle. By dividing the magnetometer output by the maximum possible output (calibrated strength of the field), we can find these cosines. By taking many measurements over a spin cycle, we can develop a system of equations that contains more equations than unknowns. The next step in the process requires knowledge of the roll angle about the spin axis. Any of the possible spin axes could produce the measurement sequence. Note that the zero value of the magnetic roll angle is unique for each candidate spin axis.

If the magnetometer value is known and associated with a magnetic roll angle, a given spin axis will be defined. Conley and Patton [1] suggest using a configuration that allows a measurement induced by the sun's energy. Using the solar pointing angle, they show that it is possible to solve for the Euler angle about the spin axis in terms of the other two Euler rotations. This measurement fixes the ray on which the spin axis must lie. The magnetometer measurements then allow the spin axis to be adjusted along the ray defined by the first measurement. This is a recursive process in which subsequent adjustments are almost orthogonal (one along a radius and the other along the circumference). Conley and Patton do not discuss the issue associated with errors in the orientation of the magnetometer leading to errors associated with the spin axis and vice versa. With their procedure, knowledge of one is assumed to find the other in a recursive algorithm. They demonstrated that this procedure converged for the problem they investigated.

A closed form solution can be devised, given the existence of two known directions (solar and magnetic fields) and the projection of a third desired direction (spin axis) onto the two known directions. The problem described in Conley and Patton fits this description. A verbal description of the implementation of a closed form solution follows. First, assume that the directions are normalized so that each is represented by a vector to the unit sphere. For each measurement, the desired direction will be on a cone determined by the known direction and the projection (or measurement). The circle formed by the cone and the unit sphere is in a plane that is normal to the known direction and contains the point along the known direction that is in proportion to the projection or inner product. It is possible to describe this plane with an equation that uses the known direction and interprets the measurement as an inner product.

A second plane, based on the other direction, can also be formulated as an equation. The intersection of these two planes forms a line, and finally, the intersection of this line with the unit sphere yields two candidate directions that fulfill the conditions. One candidate solution will be associated with increasing measurements and one with decreasing measurements. The solution based on this requires multiplication, addition, and only one square root operation. Note that if the two known directions are collinear, the planes will be parallel and no solution will exist. As the known directions approach collinearity, the solution becomes unstable.

A simulation to observe the behavior of the solution was devised. Randomly selected directions are chosen for the two known directions, and a third randomly chosen direction was chosen as the axis of interest. Noise was added to the projection of the axis of interest on each of the two known axes, and this result was sent to a routine that is based on the previously described solution. An error in the projection value affects the equation of the plane by moving it up or down along the direction to which it is orthogonal. For planes with almost collinear normals, a small change in the separation between the planes can have a large influence on their intersection. The difference between the solution and the desired direction in degrees is reported as the system error. Normally, distributed input errors with standard deviations of 0.005 radian or 0.2865 degree were used. System errors are displayed in Figure 4; the percentage of observations below a system error value is displayed.

This type of plot is referred to as a quantile plot. A quantile plot is the graph of the percentage of the data that is less than a given value; the median will occur at the abscissa value of 50. It is apparent that in 90% of the cases, the error is less than 2.5 degrees. In addition to the system error (the dependent variable), the inner product of the known axes and the determinant of all three vectors were reported as independent variables. Also, an indicator variable was used to indicate that the real part of a complex solution was used. Complex solutions

occur as the closest point of approach of an external line to the unit sphere. If two vectors are nearly collinear, their inner product will be close to 1. The determinant of three vectors indicates the volume they form; if this volume is close to 0, the vectors are nearly collinear. When the norms of each vector are 1, the volume can never exceed 1.

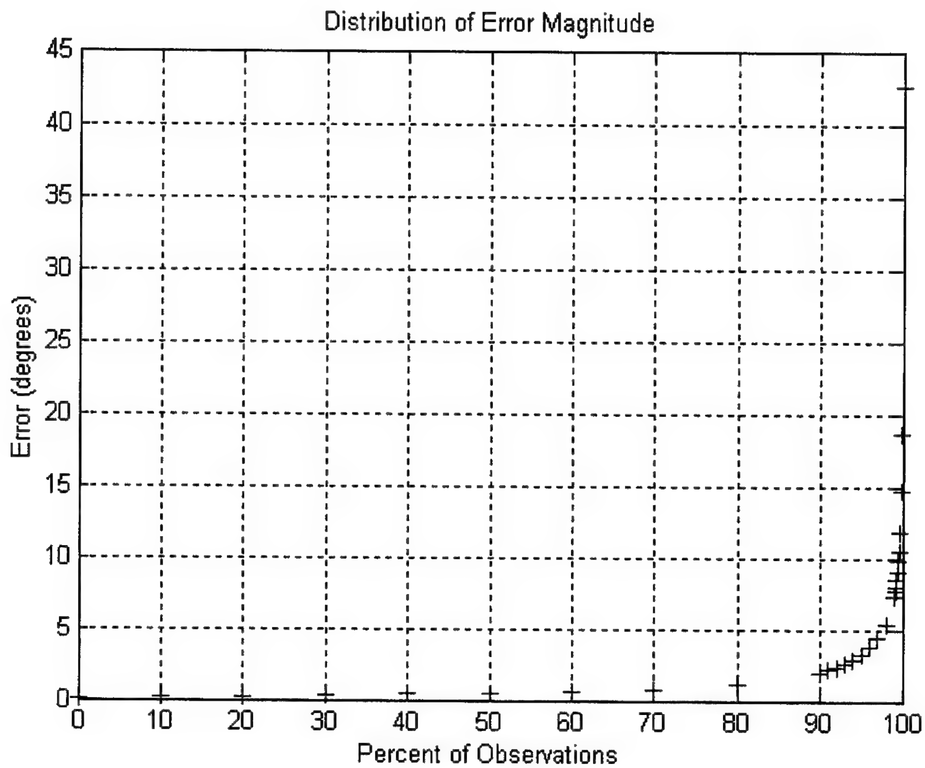

Figure 4. Error Magnitude of Complete Solution.

To make the two measures of collinearity commensurate, the determinant was subtracted from 1 so that values of 1 would represent collinear input sets, and values close to 1 would represent directions that were nearly collinear. Figure 5 shows the error plotted against the inner product with complex solutions in green. The error increases as the inner product approaches 1; also, the complex solutions are distributed throughout the distribution. Figure 6 shows the determinant measure with complex solutions in green. As 1-d increases, the maximum error slowly increases until the independent variable passes 0.8, after which, the increase accelerates. The determinant-based measure is seen to be a better indicator of the overall system error than the inner product; however, in a typical situation, only two axes are known. The determinant measure shows that all complex solutions occur when the collinearity of the three directions is high. The linear lower boundary for the complex solutions can be understood as the error level needed to force the solution to become complex for the collinearity indicator.

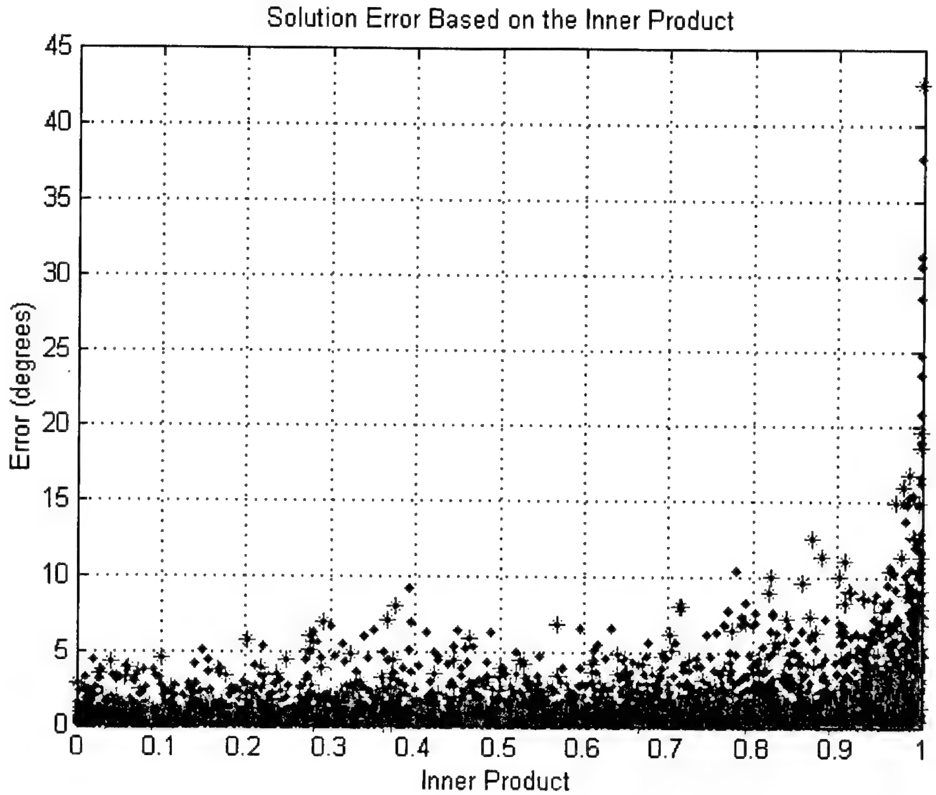

Figure 5. Relationship Between Inner Product and Error.

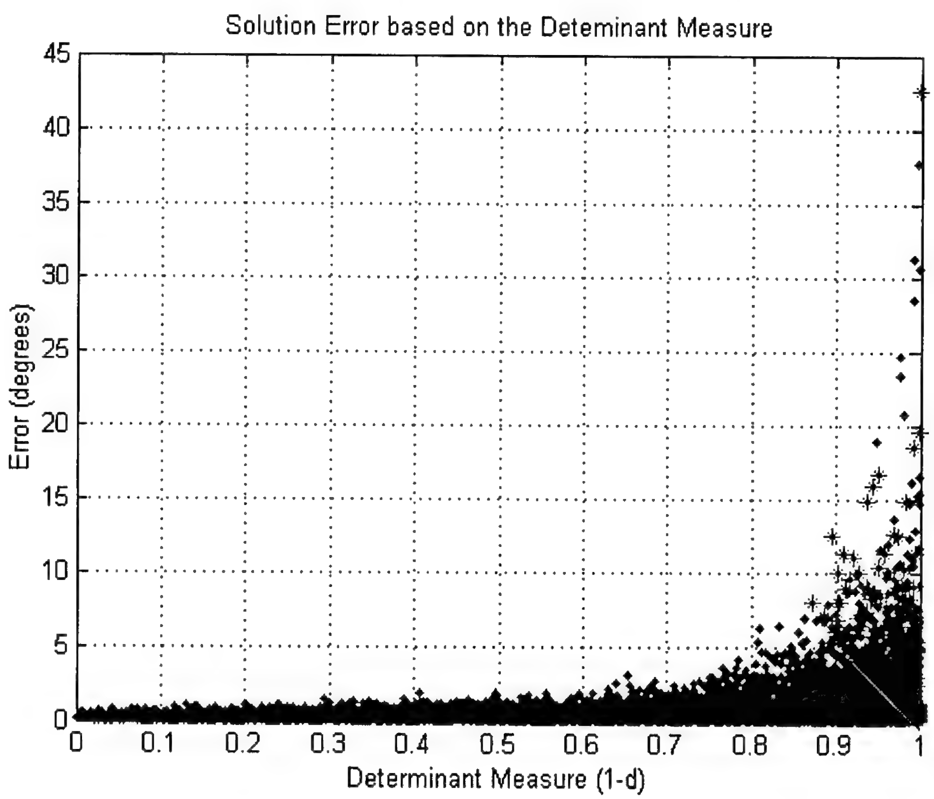

Figure 6. Relationship Between Determinant Measure and Error.

A model of system error could be made with the degree of collinearity as the predictive variable. If the system becomes collinear, then the system error would be infinite. From Figure 6, it can be seen that the reciprocal of the determinant (d, not [1-d]) could be used to predict an upper boundary on system error. The numerator could be adjusted to fit different levels of input noise. A model of this type would allow the system engineer or experiment coordinator to predict system performance for known geometries. Desirable geometries occur when the determinant is close to 1. Geometries where the determinant is close to 0 should be avoided, since large system errors will dominate system performance.

5. Solutions That Use Assumptions

In some situations, it is possible to use domain knowledge about the situation to find the attitude of the system. Specific knowledge about system behavior can be used to develop algorithms. For example, if the change in the spin axis is known to have a restriction, this knowledge can be used to orient the system. For an artillery projectile, we know that the change in the spin axis takes place mainly in the plane defined by the spin axis and gravity. This could be used in conjunction with magnetic field information to orient the system. If the magnetic field vector is parallel to the surface of the earth, then when the magnetometer reaches its maximum peak-to-peak reading, the spin axis is also parallel to the surface of the earth. This would yield two candidate systems, one of which could be eliminated. If the magnetometer is properly calibrated, the angle to the magnetic field and the attitude of the spin axis would also be known. Assuming that changes in the spin axis occur in the plane formed by the spin axis and gravity will enable computations based on the differential peak-to-peak voltages to orient the spin axis.

For an individual magnetometer spinning about an axis, the maximum and minimum readings will occur when the device is in the plane of the magnetic field and the spin axis (equal readings indicate that the spin axis is aligned with the magnetic field). These readings, coupled with knowledge of the magnetic field and an estimate of the spin axis, can be used to orient the system.

5.1 Software

The software used to evaluate formula performance is available as a MATLAB® toolbox. Eighteen routines were used in the analysis and solution of formulae presented in this report. This package can be used to evaluate other questions related to MAGSONDE data processing.

6. Conclusions

The methods discussed in this report can be used to increase the uses of magnetometers and to calibrate magnetometers. As aids to attitude estimation, magnetometers offer a way to determine a restriction of the body-fixed coordinate system. Knowledge of the cone on which the spin axis must lie, coupled with a second restriction, can lead to a direct estimate of a system's attitude.

Error analyses that were completed for the individual procedures indicate the quality of the estimate for various noise levels and parts of the magnetic roll cycle. More comprehensive models could be developed for the noise level/roll angle parameter space to provide the designer with a tool that indicates expected performance of a magnetometer processing method. The software developed for this study can be used to investigate individual cases.

These new formulae could be used in conjunction with angular rate sensors to improve their performance. Providing an independent observation of attitude to a Kalman filter would make it possible to estimate bias and scale factor errors for angular rate sensors.

The ability to estimate the angle between the spin axis and the magnetic field at any point in time will lead to real-time estimation of attitude. This increases the possibility of on-board navigation for spinning bodies.

References

1. Conley, J.M., and R.B. Patton, "An Attitude Determining System For Spinning Rockets," BRL R-1291, U.S. Army Ballisitic Research Laboratory, Aberdeen Proving Ground, MD, December 1965.

2. Harkins, T.E., and D.J. Hepner, "MAGSONDE: A Device for Making Angular Measurements on Spinning Projectiles With Magnetic Sensors," ARL-TR-2310, U.S. Army Research Laboratory, Aberdeen Proving Ground, MD, December 2000.

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Magnetometers offer measurements that define the angle to the magnetic field. Knowledge of this restricts the axis of interest to lie on a cone centered on the magnetic field direction with an angle proportional to the measurement of magnetic field strength. With this restriction, formulae are derived for finding the relationship between the magnetic field pointing vector, the spin axis, and the magnitude of the magnetic field. Methods based on both calibrated and uncalibrated magnetometers are discussed. These formulae are also useful for the calibration of magnetometers.

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